"Kernel methods in machine learning"
Homework 1
Due on 02/01/2023, 3pm

Exercice 1.
Let K_1 and K_2 be two positive definite kernels on a set $\mathcal{X}$ with corresponding RKHS's $\mathcal{H}_1$ and $\mathcal{H}_2$, and α, β two positive scalars.

1. Show that $\alpha K_1 + \beta K_2$ is positive definite.

2. Express the norm of the RKHS $\mathcal{H}$ in terms of the norms of both RKHS $\mathcal{H}_1$ and $\mathcal{H}_2$ and describe $\mathcal{H}$ in terms of elements in $\mathcal{H}_1$ and $\mathcal{H}_2$.

Exercice 2.
Let $\mathcal{X}$ be a set and $\mathcal{F}$ be a Hilbert space. Let $\Psi : \mathcal{X} \to \mathcal{F}$, and $K : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ be:

$$\forall x, x' \in \mathcal{X}, \quad K(x, x') = \langle \Psi(x), \Psi(x') \rangle_{\mathcal{F}} .$$

1. Show that K is a positive definite kernel on $\mathcal{X}$.

2. Express the norm of the RKHS $\mathcal{H}$ in terms of the norm in $\mathcal{F}$ and describe $\mathcal{H}$ in terms of elements in $\mathcal{F}$.